Name of the Course: Linear Algebra

Syllabus:

Unit I

Symmetric and Skew symmetric matrices - Hermitian and Skew Hermitian Matrices – Orthogonal and Unitary matrices - Rank of matrix - Eigen values and Eigen vectors of Linear operators – Cayley Hamilton theorem - Solutions of Homogeneous linear equations – Solutions of non homogenuous linear equations.

Section: 1.3 Orthogonal and Unitary matrices

Definition 1.3.1: Orthogonal Matrix

A real matrix P of order n is called an unitary matrix is it satisfies the condition $PP^T = I_n$.

Problem 1.3.2 : Identify the type of the matrix $P = \begin{pmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{pmatrix}$.

$$PP^T = \begin{pmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{pmatrix}\begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix}$$
$$= \begin{pmatrix} cos^2\alpha + sin^2\alpha & -sin\alpha\, cos\alpha + sin\alpha\, cos\alpha \\ -sin\alpha\, cos\alpha + sin\alpha\, cos\alpha & cos^2\alpha + sin^2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

Since $PP^T = I_2$, the given matrix is orthogonal.

Definition 1.3.3: Unitary Matrix

A Complex matrix U of order n is called an unitary matrix is it satisfies the condition $UU^* = I_n$

Problem 1.3.4 : Show that $U = \frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix}$ is unitary matrix.

Solution:

Given that $U = \frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix} \Rightarrow U^* = \frac{1}{5}\begin{pmatrix} -1-2i & 2+4i \\ -4+2i & -2+i \end{pmatrix}$

Also $UU^* = \frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix}\frac{1}{5}\begin{pmatrix} -1-2i & 2+4i \\ -4+2i & -2+i \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 25 & 0 \\ 0i & 25 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$

Since $UU^* = I_2$, the given matrix is Unitary Matrix